



Hypothetical Learning Trajectory Berbasis Realistics Mathematics Education

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Abstrak

Tujuan penelitian ini adalah untuk menghasilkan *Hypothetical Learning Trajectory* (HLT) materi penjumlahan pecahan berpenyebut sama menggunakan konteks löma makanan khas daerah Nias. Pendekatan pembelajaran yang digunakan adalah *Realistics Mathematics Education* (RME). Metode penelitian yang digunakan adalah penelitian desain. Subjek penelitian adalah tiga siswa kelas IV SDN 071104 Botohilitano dengan tingkat kemampuan heterogen yakni berkemampuan rendah sedang dan tinggi. Penelitian difokuskan untuk merancang HLT yang di validasi, di revisi dan diujicobakan secara *one-to-one* kepada peserta didik. Pengumpulan data dilakukan dengan mengumpulkan hasil pengerjaan LKPD yang diberikan, dan mewawancarai siswa. Analisis retrospektif terhadap pelaksanaan *one to one* menunjukkan bahwa penggunaan konteks instrumen yang digunakan berupa HLT dan lembar kerja peserta didik dengan menggunakan pendekatan RME melalui studi literature. Data yang dikumpulkan di analisis secara kualitatif. Berdasarkan hasil analisis diperoleh hasil bahwa peserta didik dengan kemampuan sedang dan tinggi dapat memahami materi cukup baik, sedangkan peserta didik kemampuan rendah mengalami kesulitan dalam memahami masalah yang diberikan.

Kata Kunci: Hypothetical Learning Trajectory, pecahan, RME

Abstract

The purpose of this research was to produce a Hypothetical Learning Trajectory (HLT) for the addition of fractions with the same denominator using the context of löma typical food of Nias region. The learning approach used was Realistics Mathematics Education (RME). The research method used was design research. The research subjects were three fourth grade students of SDN 071104 Botohilitano with heterogeneous ability levels, namely low, medium and high ability. The research focused on designing HLT which was validated, revised and tested one-to-one with students. Data collection was carried out by collecting the results of the NLPD given, and interviewing students. Retrospective analysis of the one to one implementation showed that the context of the instrument used in the form of HLT and learner worksheets using the RME approach through literature study. The data collected was analyzed qualitatively. Based on the results of the analysis, it was found that students with medium and high abilities could understand the material quite well, while low ability students had difficulty in understanding the problems given.

Keywords: Hypothetical Learning Trajectory, fracion, RME

Introduction

The development of science and technology in the 21st century significantly impacted the direction of change in the world of education, especially the view of the implementation of education, which was previously only based on three achievements, namely reading, writing and arithmetic. Meanwhile, 21st-century education emphasizes student-centred learning and developing skills needed in the future. Efforts are made, among others, through curriculum development, educational outcomes and the development of learning methods, one of which requires students to be skilled in problem-solving (Dakhi et al., 2020).

Mathematics is a science with a hierarchical object of study; a specific math topic will be the prerequisite for the next topic. Learn a new math topic, students' learning experience will affect the learning process. A specific material will be mastered by learners well when they can understand the prerequisite material well. With these learner characteristics, the teacher can teach the material if the teacher has an overview of the possible paths that learners go through to achieve the learning objectives. Therefore, teachers must know the learning flow of students during learning.

Mathematics learning that has recently been widely discussed is Realistic Mathematics Education (RME) or Realistic Mathematics Learning (PMR). RME is known as an approach that has been successful in the Netherlands. The idea of a realistic approach to learning mathematics is popular in the Netherlands; many developed countries have used a new approach, namely the realistic approach. Realistic mathematics is determined mainly by Freudenthal's view of mathematics. Two of his essential views are 'mathematics must be connected to reality and mathematics as human activity' (Treffers, 1987). First, mathematics must be close to learners and relevant to

everyday situations. Secondly, he emphasized that mathematics is a human activity, so students should be allowed to learn to do activities on all topics in mathematics.

Learning that emphasizes allowing students to discover mathematical concepts requires an approach, namely the RME (Realistic Mathematics Education) approach. This RME approach will create an atmosphere of meaningful learning and improve students' problem-solving skills. This RME approach, recognized by Dutch scientists that students who do mathematics learning with RME have higher scores than students who learn through traditional approaches in terms of numeracy skills, more specifically in applications. Research shows that applying the RME approach provides better results than traditional learning (Ismail et al., 2022; Zagoto, 2018). Students must not only understand the concepts relevant to the problem that is the centre of attention but also gain learning experiences related to skills in mathematical problem-solving (Budiyono, Kusumaningsih & Albab, 2019; Sarumaha, Harefa & Zagoto, 2018). Learning that uses the RME approach is one of them by using a context easily imagined by students in real life. Using contexts in the student's environment makes learning more meaningful; besides that, learning will also provide experience to students in everyday life. To support the learning context in the RME approach, the context given is about local culture. The selection of local culture is based on the fact that education must play its role in cultural development (Freudenthal, 1991).

RME-based mathematics learning involves students in learning so that they not only use specific formulas and procedures to solve problems but more than that, students are faced with several contexts of mathematical problems related to students' real lives (Afriansyah, 2017; Zagoto & Dakhi, 2018). To direct such learning conditions and situations,

developing a learning design through a learning flow that links mathematical concepts with everyday problems through several activities is necessary (Zagoto, Yarni & Dakhi, 2019). The learning design in question is a learning trajectory known as a hypothetical learning trajectory (HLT).

The learning design will be contained in the RME-based HLT. The presentation of the HLT pays attention to the characteristics of RME, which include: (a) the use of context, (b) using models, bridging with vertical instruments, (c) student contributions, (d) interactivity, and (e) linkages. This designed HLT contains (1) learning objectives; (2) activities; (3) prediction of students' thinking. Some of the things contained in the teacher's book are the cover of the teacher's book, which contains the identity of the book, realistic pictures of problems that occur in South Nias contained in the teacher's book, a preface for the teacher, a table of contents, a concept map, learning objectives, learning tools and materials, activities, time allocation, learning planning, getting to know the teaching material, student activities, let us practice, let us conclude, homework, assessment of student abilities, comments and solutions. While in the student book, namely the cover of the student book, which contains the identity of the book, and realistic pictures of the problems that occur in South Nias which are contained in the student book, a preface for students, a table of contents, a concept map, learning objectives, and contextual problems; let us practice, homework, assessment place, and let's conclude. This book can help students find the concept of the material because the problems given are adapted to students' real life.

An HLT is always prepared by teachers based on the thought of possible difficulties or obstacles experienced by students so that the best learning outcomes can be achieved (Gravemeijer & Cobb, 2013). It can be seen in the thinking and planning that takes place in teaching,

including spontaneous responses to students' thinking. In addition, before learning takes place, teachers already know the difficulties and obstacles in the learning flow which students are likely to experience. Hence, teachers need to think in advance to prepare appropriate materials, methods, and presentation strategies, so that students are always on an alternative trajectory, or HLT, following expectations so that they can achieve the desired learning objectives (Simon, 1995).

Method

The method used in this research is design research by Gravemeijer and Cobb (Akker et al., 2018). Design research is conducted by designing and learning about fraction addition using the RME approach. The instruments of this research are Hypothetical Learning Trajectory (HLT) and students' worksheets. Design research proposed by Gravemeijer and Cobb is focused on developing the order of presentation of material in the process of learning mathematics. At this stage, the researcher tested the HLT designed for three fourth-grade SDN 071104 Botohilitano who came from low, medium and high ability students. Then, at the retrospective stage of the experimental results obtained in the one-to-one activity stage.

Result And Discussion

Result

Based on the LKPD that students do, the results of solving according to their abilities are as follows:

Low-ability learners. Based on the LKPD that students do, students need help to answer problems correctly; writing the part that Wati eats should be in the form of fractions.

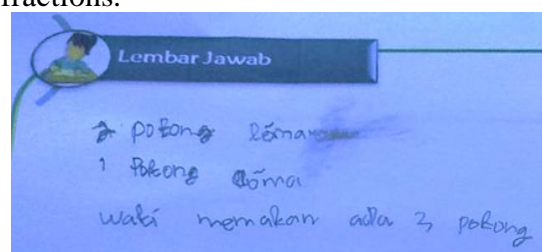


Figure 1: Low-Ability Learners' Answers

From the results of the student's answers, then the teacher tries to conduct a brief interview about the student's answers.

Teacher: Ananda, why is your answer like this?

Student: Because Wati ate two pieces and one piece, so three pieces.

Teacher: Why don't you write the answer in fraction form?

Learner: Sorry, ma'am, I am confused.

Teacher: From the problem, what is known?

Learner: Lemang is divided by eight by Mom.

Teacher: That is good. Then, for each part or piece, what is the fractional value?

Learner : $\frac{8}{8}$, ma'am

Teacher: Good, the total value is $\frac{8}{8}$, so what is the value per part or piece in fractions?

Learner : $\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}$, and $\frac{1}{8}$.

Teacher: That is smart. When answering it, you should use the fraction value, 1 piece is $\frac{1}{8}$.

Student: Yes, ma'am, I start to understand now.

Furthermore, the teacher invites learners to understand again and guides students on how to solve the problems given previously so that it is obtained that $\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$.

Learners with moderate ability. Based on the LKPD that students do, students can also answer the problem correctly by making a picture of the löma piece. Students can understand the concept of 3 parts and write it in fraction form. However, the addition process of 2 pieces and one piece should be better written in fraction form.

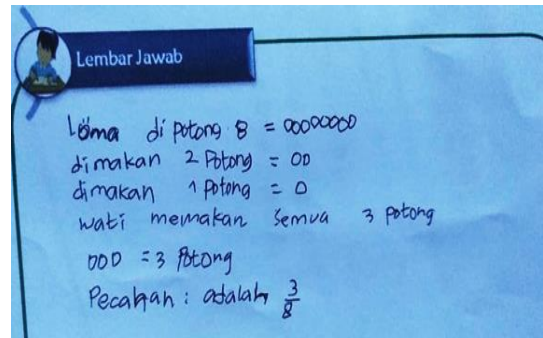


Figure 2. Answers of Medium Ability Learners

From the results of the student's answers, then the teacher tries to conduct a brief interview about the student's answers.

Teacher: Ananda, why is your answer like that?

Learner: Because Wati ate 2 pieces of löma and mom gave her another 1 piece, there are 3 pieces total.

Teacher: Why don't you write the answer in fraction form?

Learner: My paper directly adds 2 pieces and 1 piece ma'am, then I make the result in fractions, but I understand, ma'am.

Teacher: Okay. From the problem, what is known?

Student: Mom cooked Loma and cut it into 8 pieces.

Teacher: Well, good. Then, for each part or piece, what is the fractional value?

Learner: 1 piece is worth $\frac{1}{8}$, ma'am, because it is divided into eight, so the fraction is per 8.

Teacher : That is smart. When answering it, you should use the fraction value, 1 piece is $\frac{1}{8}$. And so on until the calculation of the amount of loma that Wati ate.

Student: Yes, ma'am.

High-ability learners. Based on the LKPD that students do, students can answer the questions correctly, even though there is an explanation in the answer.

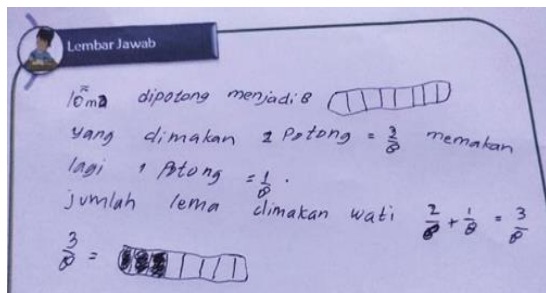


Figure 3: High-Ability Learners' Answers

Teacher: Ananda, why is your answer like that?

Student: Because I understand the way to add like this better.

Teacher: Okay. From the problem, what is known?

Learner: Mom cooked löma and cut it into 8 pieces.

Teacher: Well, good. Then, for each part or piece, what is the fractional value?

Learner: 1 piece is worth $\frac{1}{8}$, ma'am, because the total is eight pieces or the fraction value is $\frac{8}{8}$.

Teacher: That is smart. When answering it, you should use the fraction value, 1 piece is $\frac{1}{8}$. And so on until the calculation of the amount of löma that Wati ate.

Student: Yes, ma'am.

Discussion

The activities at this stage tested the designed learning flow by distributing the LKPD to students and HLT to teachers. LKPD was tested on three students with different academic levels, namely high, medium and low ability.

Based on the discussion above, low-ability students from the LKPD students have not understood the concept of fractions in general and adding fractions with the same denominator. Learners only immediately add up the pieces of löma that Wati eats using the addition of natural numbers or whole numbers.

Based on the discussion above, from the LKPD that students do, it appears that students have been able to find ways to

determine the sum of fractions with the same denominator. However, students from the beginning did not write down the pieces of löma that were eaten in the form of fractions. Learners only immediately write the fraction form in the final result of the addition.

High ability students, based on the discussion above, from the LKPD that students do, it appears that students can understand the concept of adding fractions with the same denominator.

So, low-ability learners need help understanding the concept of fractions and cannot write a part in fraction form. With anticipation and teacher guidance, learners began to be able to write a part in the form of a fraction, although still when performing fraction addition operations. Learners with moderate ability can understand context problems by looking for various possibilities and drawing löma pieces. Although the way of working is not detailed, the results are correct. In contrast, learners with high ability can understand contextual problems by making pictures of löma pieces and can even complete the addition operation of fractions with the same denominator because they already understand the concept well.

The implementation of learning is extended because many learners still choose to be silent when given questions about the problem being studied. So the teacher needs extra effort so students can find ways to solve the problems. The concentration of students is also reduced, so it is not easy to condition good learning. It also made the learning that was carried out less effective, especially when the weather was also raining.

Based on the activities carried out by students and by using this HLT, students can independently find the mathematical model of the addition of fractions with the same denominator. At the end of the lesson, it was emphasized again how to determine the sum of 2 or more fractions with the same denominator. What was predicted and anticipated in the HLT

design was carried out appropriately, although under the teacher's guidance and supported by interviews with students.

According to Hadi (2017), a hypothetical learning pathway (HLT) is the conjecture of a designer or a researcher regarding the possible learning pathways that occur in the classroom when designing learning. Because it is hypothetical, it is undoubtedly only sometimes true. Many are wrong because what happens in the classroom is often unexpected. After the researcher (in this case, the designer) conducts a trial, the actual learning flow is obtained, which is called the learning flow. The learning flow can be used as a new hypothetical learning flow in the following learning cycle.

Löma pieces can be expressed in terms of fraction symbols. Learners can show the Löma piece based on the fraction symbol given from the problem because, in previous learning, learners have recognized the fraction symbol. Learners with a strong concept of fractions can determine the sum of two fractions with the same denominator. By counting the number of löma pieces, learners can determine the sum of two fractions with the same denominator. The pieces of Löma that are counted are the sum of the numerators of the given fractions. Based on the sum result, learners can see and realize that the denominator of the sum of the fractions does not add up. From the activity of counting the number of Löma, it can also be seen that the overall pieces of Löma do not change or increase. With this activity, learners can discover that adding fractions with the same denominator is done by adding the fractions' numerators. The activity of solving addition problems of Löma pieces will help students find the algorithm for adding fractions with the same denominator..

Conclusion

Based on the research results and discussion, the hypothetical learning trajectory can be applied to adding

fractions with the same denominator. In one-to-one application, medium and high-ability learners can answer according to what is predicted in HLT. However, low-ability learners still need to understand the concept. However, the student begins to understand after being guided by the teacher.

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